Real Estate Principles

Lesson 19:
Real Estate Math
Solving Math Problems

Four steps

1. **Read** the question.
2. **Write** down the formula.
3. **Substitute** the numbers in the problem into the formula.
4. **Calculate** the answer.
Solving Math Problems
Using formulas

Each of these choices expresses the same formula, but in a way that lets you solve it for A, B, or C:

\[ A = B \times C \]
\[ B = A \div C \]
\[ C = A \div B \]
Isolate the unknown.

- The unknown is the element that you’re trying to determine.
- The unknown should always sit alone on one side of the equals sign.
- All the information that you already know should be on the other side.
Example: What is the length of a property that is 9,000 square feet and 100 feet wide?

• The formula for area is $A = L \times W$.

• $L$ is the unknown, so switch the formula to $L = A \div W$.

\[
L = 9,000 \div 100
\]
\[
90 = 9,000 \div 100
\]
Decimal Numbers
Converting fraction to decimal

-Calculators use only decimals, not fractions.

-If a problem contains a fraction, convert it to a decimal:
  -Divide the top number (the numerator) by the bottom number (the denominator).

\[
\begin{align*}
1/4 &= 1 \div 4 = 0.25 \\
1/3 &= 1 \div 3 = 0.333 \\
5/8 &= 5 \div 8 = 0.625
\end{align*}
\]
Decimal Numbers

Converting decimal to percentage

To convert a decimal to a percentage, move the decimal point two numbers to the right and add a percent sign.

0.02 = 2%
0.80 = 80%
1.23 = 123%
Decimal Numbers

Converting percentage to decimal

- To convert a percentage to a decimal, reverse the process:
  - Move the decimal point two numbers to the left and remove the percent sign.

  \[
  \begin{align*}
  2\% &= 0.02 \\
  80\% &= 0.8 \\
  123\% &= 1.23
  \end{align*}
  \]
## Summary
### Solving Math Problems

- **Read problem**
- **Write formula and isolate the unknown**
- **Substitute**
- **Calculate**

- **Fractions**
- **Decimal numbers**
- **Percentages**
- **Conversion**
Area Problems

Formula: $A = L \times W$

To determine the area of a rectangular or square space, use this formula:

$A = L \times W$
Area Problems

You might also be asked to factor other elements into an area problem, such as:

- cost per square foot,
- rental rate, or
- the amount of the broker’s commission.
Area Problems
Example

An office is 27 feet wide by 40 feet long. It rents for $2 per square foot per month. How much is the monthly rent?

- Part 1: Calculate area
  \[ A = 27 \text{ feet} \times 40 \text{ feet} \]
  \[ A = 1,080 \text{ square feet} \]

- Part 2: Calculate rent
  \[ \text{Rent} = 1,080 \times \$2 \]
  \[ \text{Rent} = \$2,160 \]
Some problems express area in square yards rather than square feet.

Remember: 1 square yard = 9 square feet

- 1 yard is 3 feet
- 1 square yard measures 3 feet on each side
- 3 feet × 3 feet = 9 square feet
Area Problems

Triangle formula: \( A = \frac{1}{2} B \times H \)

To determine the area of a right triangle, use this formula:

\[ A = \frac{1}{2} B \times H \]

Right triangle: a triangle with a 90\(^\circ\) angle
Area of a Triangle

- Visualize a rectangle, then cut it in half diagonally. What’s left is a right triangle.
  - If you’re finding the area of a right triangle, it doesn’t matter at what point in the formula you cut the rectangle in half.
  - In other words, any of these variations will reach the same result:
    \[
    A = \frac{1}{2} B \times H \\
    A = B \times \frac{1}{2} H \\
    A = (B \times H) \div 2
    \]
A triangular lot is 140 feet long and 50 feet wide at its base. What is the area?

- Do the calculation in any of the following ways to get the correct answer.
A triangular lot is 140 feet long and 50 feet wide at its base. What is the area?

Variation 1:

\[ A = \left( \frac{1}{2} \times 50 \right) \times 140 \]
\[ A = 25 \times 140 \]
\[ A = 3,500 \text{ sq. feet} \]
A triangular lot is 140 feet long and 50 feet wide at its base. What is the area?

**Variation 2:**

\[ A = 50 \times \left( \frac{1}{2} \times 140 \right) \]

\[ A = 50 \times 70 \]

\[ A = 3,500 \text{ sq. feet} \]
A triangular lot is 140 feet long and 50 feet wide at its base. What is the area?

**Variation 3:**

\[ A = (50 \times 140) \div 2 \]

\[ A = 7,000 \div 2 \]

\[ A = 3,500 \text{ sq. feet} \]
Area Problems

Odd shapes

To find the area of an irregular shape:

1. Divide the figure up into squares, rectangles, and right triangles.
2. Find the area of each of the shapes that make up the figure.
3. Add the areas together.
The lot’s western side is 60 feet long. Its northern side is 100 feet long, but its southern side is 120 feet long.

To find the area of this lot, break it into a rectangle and a triangle.
Odd Shapes

Example, continued

Area of rectangle
A = 60 × 100
A = 6,000 sq. feet
To find the length of the triangle’s base, subtract length of northern boundary from length of southern boundary.

120 – 100 = 20 feet

Area of triangle:

\[ A = \left(\frac{1}{2} \times 20\right) \times 60 \]

\[ A = 600 \text{ sq. feet} \]
Odd Shapes

Example, continued

Total area:

\[ 6,000 + 600 = 6,600 \text{ sq. feet} \]
A common mistake when working with odd shapes is to calculate the area of part of the figure twice. This can happen with a figure like this one.
Odd Shapes
Avoid counting same section twice

Here’s the wrong way to calculate the area of this lot.

\[25 \times 50 = 1,250\]
\[40 \times 20 = 800\]
\[1,250 + 800 = 2,050\]

By doing it this way, you measure the middle of the shape twice.
Odd Shapes

Avoid counting same section twice

Avoid the problem by breaking the shape down like this instead.

Find height of smaller rectangle by subtracting height of top rectangle (25 feet) from height of the whole shape (40 feet).

\[40 - 25 = 15 \text{ feet}\]
Now calculate the area of each rectangle and add them together:

25 × 50 = 1,250 sq. ft.

20 × 15 = 300 sq. ft.

1,250 + 300 = 1,550 sq. ft.
Odd Shapes
Avoid counting same section twice

Here’s another way to break the odd shape down into rectangles correctly.

To find width of the rectangle on the right, subtract width of left rectangle from width of whole shape:
50 \(-\) 20 = 30 feet

\[
\begin{array}{c}
\text{40'} \\
\times 20'
\end{array}
\Rightarrow 800 \text{ Sq. Feet}
\]

\[
\begin{array}{c}
25' \\
\times 30'
\end{array}
\Rightarrow 750 \text{ Sq. Feet}
\]

\[
15' \\
\times 30'
\Rightarrow 450 \text{ Sq. Feet}
\]

Total: 1,550 Sq. Feet
Now calculate the area of each rectangle and add them together:

\[ 40 \times 20 = 800 \text{ sq. ft.} \]
\[ 30 \times 25 = 750 \text{ sq. ft.} \]

\[ 800 + 750 = 1,550 \text{ sq. ft.} \]
Odd Shapes
Narrative problems

- Some area problems are expressed only in narrative form, without a visual.

- In that case, draw the shape yourself and then break the shape down into rectangles and triangles.
Odd Shapes

Example

A lot’s boundary begins at a certain point and runs due south for 319 feet, then east for 426 feet, then north for 47 feet, and then back to the point of beginning.

To solve this problem, first draw the shape.
A lot’s boundary begins at a certain point and runs due south for 319 feet, then east for 426 feet, then north for 47 feet, and then back to the point of beginning.
Odd Shapes
Example, continued

Break it down into a rectangle and a triangle as shown.

Subtract 47 from 319 to find the height of the triangular portion.

319 – 47 = 272 feet
Odd Shapes
Example, continued

Calculate the area of the rectangle.

426 × 47 = 20,022 sq. ft.
Calculate the area of the triangle.

\[
\frac{1}{2} \times 426 \times 272 = 57,936 \text{ sq. feet}
\]
Odd Shapes
Example, continued

Add together the area of the rectangle and the triangle to find the lot’s total square footage.

20,022 + 57,936 = 77,958 sq. feet
Volume Problems

- **Area**: A measurement of a two-dimensional space.
- **Volume**: A measurement of a three-dimensional space.
  - Width, length, and height
  - Cubic feet instead of square feet
Volume Problems

Formula: $V = L \times W \times H$

To calculate volume, use this formula:

$$V = L \times W \times H$$

Volume = Length × Width × Height
If you see a problem that asks for cubic yards, remember that there are 27 cubic feet in a cubic yard:

$$3 \text{ feet} \times 3 \text{ feet} \times 3 \text{ feet} = 27 \text{ cubic feet}$$
Volume Problems

Example

A trailer is 40 feet long, 9 feet wide, and 7 feet high. How many cubic yards does it contain?

\[40 \times 9 \times 7 = 2,520 \text{ cubic feet}\]

\[2,520 \div 27 = 93.33 \text{ cubic yards}\]
Summary
Area and Volume

- Area of a square or rectangle: $A = L \times W$
- Area of a right triangle: $A = \frac{1}{2} B \times H$
- Divide odd shapes into squares, rectangles, and triangles
- Volume: $V = L \times W \times H$
- Square feet, square yards, cubic feet, cubic yards
Many math problems ask you to find a certain percentage of another number. This means that you will need to multiply the percentage by that other number.
Percentage Problems
Working with percentages

- Percentage problems usually require you to change percentages into decimals and/or decimals into percentages.

- Example: What is 85% of $150,000?
  
  \[ 0.85 \times \$150,000 = \$127,500 \]
One common example of a percentage problem is calculating a commission.

Example: A home sells for $300,000. The listing broker is paid a 6% commission on the sales price. The salesperson is entitled to 60% of that commission. How much is the salesperson’s share?

\[ \$300,000 \times .06 = \$18,000 \]
\[ \$18,000 \times .60 = \$10,800 \]
Percentage Problems

Formula: $W \times \% = P$

- Basic formula for solving percentage problems:

  Whole $\times$ Percentage = Part

  $W \times \% = P$
Percentage Problems

Formula: $W \times \% = P$

- The “whole” is the larger figure, such as the property’s sale price.
- The “part” is the smaller figure, such as the commission owed.
- Depending on the problem, the “percentage” may be referred to as the “rate.”
  - Examples: a 7% commission rate, a 5% interest rate, a 10% rate of return
Percentage Problems
Interest and profit problems

Note that you’ll also use the percentage formula when you’re asked to calculate interest or profit.

Example: A lender makes an interest-only loan of $140,000. The interest rate is 6.5%. How much is the annual interest?

\[ W \times \% = P \]

\[ $140,000 \times .065 = $9,100 \]
Example: An investor makes an $85,000 investment. She receives a 12% annual return on her investment. What is the amount of her profit?

\[ W \times \% = P \]

\[ \$85,000 \times 0.12 = \$10,200 \]
If you need to determine the percentage (the rate) or the amount of the whole, rearrange the formula to isolate the unknown on one side of the equals sign.

\[ A = B \times C \quad \quad \quad P = W \times \% \]
\[ A \div B = C \quad \quad \quad P \div W = \% \]
\[ A \div C = B \quad \quad \quad P \div \% = W \]
Percentage Problems
Finding the percentage or rate

Example: An investor makes an $85,000 investment and receives a $10,200 return. What is the rate of return?

\[ \frac{P}{W} = \% \]

\[ \frac{10,200}{85,000} = .12 \text{ (or 12\%)} \]
Percentage Problems
Finding the whole

Example: An investor receives a $10,200 return on her investment. This is a 12% return on her investment. How much did she invest?

\[
P \div \% = W
\]

\[
$10,200 \div .12 = $85,000
\]
Percentage Problems

Multiply or divide?

Knowing when to divide or to multiply can be the hardest part of solving percentage problems. Rule of thumb:

- If the missing element is the part (the smaller number), it’s a multiplication problem.
- If the missing element is either the whole (the larger number) or the percentage, it’s a division problem.
Example: A lender makes an interest-only loan of $140,000. The annual interest is $9,100. What is the interest rate?

You know the part (the interest) and the whole (the loan amount). The percentage (the interest rate) is the missing element, so this is a division problem.

\[
P \div W = \% \\
$9,100 \div $140,000 = .07 = 7\%
\]
Example: An investor receives a $10,200 return on her investment. This is a 12% return on her investment. How much did she invest?

You know the part (the profit) and the percentage (the rate of return). The whole (the total investment) is the missing element, so this is a division problem.

\[ P \div \% = W \]

\[ \$10,200 \div .12 = \$85,000 \]
Multiply or Divide?

Finding the part

Example: A home sells for $300,000. The listing broker is paid a 6% commission on the sales price. The salesperson is entitled to 60% of that commission. How much is the salesperson’s share?

You know the whole (the sale price) and the rate. The part (the commission) is the missing element, so this is a multiplication problem.

\[ W \times \% = P \]

\[ $300,000 \times .06 = $18,000 \]
Summary

Percentage Problems

- Percentage formula:
  \[ \text{Whole} \times \text{Percentage (Rate)} = \text{Part} \]
  \[ W \times \% = P \]
  \[ P \div W = \% \]
  \[ P \div \% = W \]

- Types of percentage problems: commission problems, interest problems, and profit problems.
You’ve already learned how to solve interest problems where the interest is given as an annual figure.

Let’s look at how to solve problems where interest is given in semiannual, quarterly, or monthly installments.

- In each case, the first step is to convert the interest into an annual figure.
Example: A real estate loan calls for semiannual interest-only payments of $3,250. The interest rate is 9%. What is the loan amount?

Semiannual: two payments per year.

$3,250 \times 2 = $6,500 annual interest

You know the part (the interest) and the rate. You need to find the whole (the loan amount).

\[ P \div \% = W. \]

\[ $6,500 \div .09 = $72,222.22 \]
Example: A real estate loan calls for quarterly interest-only payments of $2,371.88. The loan balance is $115,000. What is the interest rate?

Quarterly: 4 payments per year.

\[ \text{Interest} = \text{Quarterly Payment} \times 4 = $9,487.52 \]  

You know the part (the interest) and the whole (the loan amount). You need to find the rate.

\[ \text{Rate} = \frac{\text{Interest}}{\text{Loan Amount}} = \% \]

\[ \frac{$9,487.52}{$115,000} = .0825 \text{ or } 8.25\% \]
Loan Problems
Monthly interest

Example: The interest portion of a loan’s monthly payment is $517.50. The loan balance is $92,000. What is the interest rate?

Monthly: 12 payments per year

$517.50 \times 12 = $6,210 (annual interest)

You know the part (the interest) and the whole (the loan amount). You need to find the rate.

\[ P \div W = \% \]

\[ $6,210 \div $92,000 = .0675 \text{ or } 6.75\% \]
Loan Problems
Amortization

- Some problems will tell you the interest portion of a monthly payment and ask you to determine the loan’s current principal balance.

- Solve these in the same way as the problems just discussed.
Example: The interest portion of a loan’s monthly payment is $256.67. The interest rate is 7%. What is the loan balance prior to the fifth payment?

$256.67 \times 12 = $3,080.04 \text{ (annual interest)}$

You know the part (the interest) and the rate, and you need to find the whole (the loan balance).

\[ P \div \% = W \]

\[ $3,080 \div .07 = $44,000 \]
Loan Problems
Amortization

- Some problems may tell you the monthly principal and interest payment (instead of just the interest portion of the monthly payment).

- These require several additional steps.
Example: The balance of a loan is $96,000. The interest rate is 8%. The monthly principal and interest payment for a loan is $704.41. How much will this payment reduce the loan balance?
Loan balance: $96,000  Monthly P&I: $704.41
Interest rate: 8%

- Step 1: Calculate the annual interest.
  \[ W \times \% = P \]
  \[ $96,000 \times .08 = $7,680 \text{ (annual interest)} \]

- Step 2: Calculate the monthly interest.
  \[ $7,680 \div 12 = $640 \]
Loan Problems

Amortization

Loan balance: $96,000  Monthly P&I: $704.41
Interest rate: 8%  Monthly interest: $640

- Step 3: Subtract monthly interest from total monthly payment to determine monthly principal.
  
  $704.41 – $640 = $64.41

- Step 4: Subtract monthly principal from loan balance.
  
  $96,000 – $64.41 = $95,935.59
You might see a question like this where you’re asked how much the second or third payment will reduce the loan balance.

In that case, you would calculate the first payment’s effect and then repeat the four steps again, using the new balance.
Step 1: $95,935.59 \times 0.08 = $7,674.85
Step 2: $7,674.85 ÷ 12 = $639.57
Step 3: $704.41 – $639.57 = $64.84
Step 4: $95,935.59 – $64.84 = $95,870.75

The second payment would reduce the loan balance to $95,870.75.

To see how much the third payment would reduce the loan balance, you’d repeat the four steps yet again.
Summary

Loan Problems

- Use the percentage formula for loan problems.  
  \[ \text{Whole} \times \text{Percentage (Rate)} = \text{Part} \]

- Convert semiannual, quarterly, or monthly interest into annual interest before substituting numbers into formula.

- Amortization problems ask you to find a loan’s principal balance.
Profit or Loss Problems

Another common type of percentage problem involves a property owner’s profit or loss over a period of time.

- Here the “whole” is the property’s value at an earlier point (which we’ll call Then).
- The “part” is the property’s value at a later point (which we’ll call Now).
Profit or Loss Problems

“Then” and “Now” formula

- The easiest way to approach these problems is by using this modification of the percentage formula:
  
  \[ \text{Then} \times \text{Percentage} = \text{Now} \]

- Of course, this can be changed to:
  
  \[ \text{Now} \div \text{Percentage} = \text{Then} \]
  
  \[ \text{Now} \div \text{Then} = \text{Percentage} \]
Profit or Loss Problems

Calculating a loss

Example: A seller sells her house for $220,000, which represents a 30% loss. How much did she originally pay for the house?

- You know the Now value and the percentage of the loss.
- You need to find the Then value (the original value of the house).
- Rearrange the basic formula to isolate Then:

\[
\text{Now} ÷ \text{Percentage} = \text{Then}
\]
Profit or Loss Problems

Calculating a loss

Now ÷ Percentage = Then
$220,000 ÷ .70 = $314,286

The key to solving this problem is choosing the correct percentage to put into the formula.

• Here the correct percentage is 70%, not 30%.
• The house didn’t sell for 30% of its original value. It sold for 30% less than its original value.

100% – 30% = 70%
Profit or Loss Problems
Calculating a loss

When dealing with a loss, you can determine the rate using this formula:
100% – Percentage Lost = Percentage Received

It’s the percentage received that must be used in the formula.
Profit or Loss Problems

Calculating a gain

To calculate a gain in value, add the percentage gained to 100% find the percentage received:

\[100\% + \text{Percentage Gained} = \text{Percentage Received}\]

Returning to the example, if the sale had resulted in a 30% profit instead of a 30% loss, that would mean the house sold for 130% of what the seller originally paid for it:

\[100\% + 30\% = 130\%\]
Example: A seller sells her house for $220,000, which represents a 30% gain. How much did she originally pay for the house?

\[ \frac{220,000}{1.30} = 169,231 \]

Now \( \div \) Percentage Received = Then
Profit or Loss Problems

Calculating a gain

- Note that if a seller sells a house for 130% of what she paid for it, she didn’t make a 130% profit.

- She received 100% of what she paid, plus 30%. She received a 30% profit.
A profit or loss problem may also be expressed in terms of appreciation or depreciation.

If so, the problem is solved the same way as an ordinary profit and loss problem.
You may see problems where you’re told how much a property appreciated or depreciated per year over several years.

This requires you to repeat the same calculation for each year.
Example: A property is currently worth $220,000. It has depreciated four and a half percent per year for the past five years. What was the property worth five years ago?
The house is losing value, so first subtract the rate of loss from 100%.

\[
100\% - 4.5\% = 95.5\%, \text{ or } .955
\]

You know the Now value and the rate. The missing element is the Then value:

\[
\text{Now} \div \text{Percentage} = \text{Then}
\]

\[
$220,000 \div .955 = $230,366.49
\]

The house was worth $230,366 one year ago.
Now repeat the calculation four more times, to determine how much the house was worth five years ago:

$230,366 ÷ .955 = $241,221 (value 2 years ago)
$241,221 ÷ .955 = $252,587 (value 3 years ago)
$252,587 ÷ .955 = $264,489 (value 4 years ago)
$264,489 ÷ .955 = $276,952 (value 5 years ago)
If you’re told that a property gained value at a particular rate over several years, you’ll use the same process.

The difference is that you’ll need to add the rate of change to 100%, instead of subtracting it from 100%.
Example: A property is currently worth $380,000. It has appreciated in value 4% per year for the last four years. What was it worth four years ago?
Profit or Loss Problems

Compound appreciation

- Add the rate of appreciation to 100%.
  \[ 100\% + 4\% = 104\%, \text{ or } 1.04 \]
- You know the Now value and the rate of change, so use the formula \( \text{Now} \div \text{Percentage} = \text{Then} \).

\[ \begin{align*}
  \$380,000 \div 1.04 &= \$365,385 \text{ (value 1 year ago)} \\
  \$365,385 \div 1.04 &= \$351,332 \text{ (value 2 years ago)} \\
  \$351,332 \div 1.04 &= \$337,819 \text{ (value 3 years ago)} \\
  \$337,819 \div 1.04 &= \$324,826 \text{ (value 4 years ago)}
\end{align*} \]
Summary

Profit or Loss Problems

- Then $\times$ Percentage = Now
- To find the percentage received:
  - If there’s been a loss in value, subtract the rate of change from 100%.
  - If there’s been a gain (a profit), add the rate of change to 100%.
- Compound appreciation and depreciation: repeat the profit or loss calculation as needed.
Capitalization Problems

Capitalization: The process used to convert a property’s income into the property’s value.

- In the appraisal of income property, the property’s value depends on its income.
  - The value is the price an investor would be willing to pay for the property.
  - The property’s annual net income is the return on the investment.
Capitalization Problems

Formula: \( V \times \% = I \)

- Capitalization problems are another type of percentage problem.
  
  Whole \( \times \) Percentage = Part

- Here the “part” is the property’s income, and the “whole” is the property’s value:
  
  \( \text{Value} \times \text{Capitalization Rate} = \text{Income} \)

  or

  \( \text{Income} \div \text{Rate} = \text{Value} \)

  or

  \( \text{Income} \div \text{Value} = \text{Rate} \)
Capitalization Problems

Capitalization rate

- The capitalization rate represents the rate of return an investor would likely want on this investment.

- An investor who wants a higher rate of return would not be willing to pay as much for the property as an investor who’s willing to accept a lower rate of return.
Example: A property generates an annual net income of $48,000. An investor wants a 12% rate of return on his investment. How much could he pay for the property and realize his desired rate of return?

\[
\text{Income} \div \text{Rate} = \text{Value}
\]

\[
$48,000 \div 0.12 = $400,000
\]

The investor could pay $400,000 for this property and realize a 12% return.
Example: An investment property has a net income of $40,375. An investor wants a 10.5% rate of return. What would the value of the property be for her?

Income ÷ Rate = Value

\[
\frac{40,375}{0.105} = \$384,524
\]

She could pay $384,524 for this property and realize a 10.5% return.
Example: An investment property is valued at $425,000 and its net income is $40,375. What is the capitalization rate?

Income ÷ Value = Rate

$40,375 ÷ $425,000 = .095, or 9.5%
Capitalization Problems
Changing the cap rate

The capitalization rate is up to the investor. It depends on how much risk he or she is willing to absorb.

- One investor might be satisfied with a 9.5% cap rate.
- Another more aggressive investor might want a 10.5% return on the same property.

Some problems ask how a property’s value will change if a different cap rate is applied.
Step 1: Calculate the property’s net income. You know the value and the rate, so use the formula Value × Rate = Income.

\[ \text{Value} \times \text{Rate} = \text{Income} \]

\[ \$450,000 \times .10 = \$45,000 \]

Step 2: Calculate the value at the higher cap rate. Income ÷ Rate = Value

\[ \frac{\text{Income}}{\text{Rate}} = \text{Value} \]

\[ \frac{\$45,000}{.11} = \$409,091 \]

The property would be worth $40,909 less at the higher cap rate.
Capitalization Problems
Changing the cap rate

Example: A property with a net income of $16,625 is valued at $190,000. If its cap rate is increased by 1%, what would its new value be?
Capitalization Problems

Changing the cap rate

- Step 1: Find the current capitalization rate.
  \[
  \text{Income} \div \text{Value} = \text{Rate}
  \]
  \[
  \frac{16,625}{190,000} = 0.0875
  \]

- Step 2: Increase the cap rate by 1%.
  \[
  8.75\% + 1\% = 9.75\%, \text{ or } 0.0975
  \]
Step 1: Find the current capitalization rate.

Income ÷ Value = Rate

$16,625 ÷ $190,000 = .0875

Step 2: Increase the cap rate by 1%.

8.75% + 1% = 9.75%, or .0975

Step 3: Calculate the new value.

Income ÷ Rate = Value.

$16,625 ÷ .0975 = $170,513

Capitalization Problems
Changing the cap rate
In some problems, you’ll be given the property’s annual gross income and a list of the operating expenses instead of the annual net income.

Before you can use the capitalization formula, you’ll have to subtract the expenses from the gross income to get the net income.
Example: A six-unit apartment building rents three units for $650 a month and three units for $550 a month. The annual operating expenses are $4,800 for utilities, $8,200 for property taxes, $1,710 for insurance, $5,360 for maintenance, and $2,600 for management fees. If the capitalization rate is 8¾%, what is the property’s value?
Step 1: Calculate the gross annual income.

$550 \times 3 \times 12 = $19,800$

$650 \times 3 \times 12 = $23,400$

$19,800 + 23,400 = $43,200$ (gross income)
Capitalization Problems
Calculating net income

- Step 2: Subtract expenses from gross income.
  
  $43,200
  -$4,800
  -$8,200
  -$1,710
  -$5,360
  -$2,600
  
  $20,530 (net income)
Step 3: Calculate the value. You know the net income and the rate, so use the formula \( \text{Income} \div \text{Rate} = \text{Value} \).

\[ \$20,530 \div .0875 = \$234,629 \]
Some problems give you the property’s operating expense ratio (OER) rather than a list of the operating expenses.

- The OER is the percentage of the gross income that goes to pay operating expenses.
- Multiply the gross income by the OER to determine the annual operating expenses. Then subtract the expenses from the gross income to determine the net income.
Capitalization Problems
Calculating net income: OER

Example: A store grosses $758,000 annually. It has an operating expense ratio of 87%. With a capitalization rate of 9¼%, what is its value?
Capitalization Problems
Calculating net income: OER

Step 1: Multiply the gross income by the OER.
$758,000 \times .87 = $659,460 \text{ (operating expenses)}$

Step 2: Subtract the expenses from gross income.
$758,000 - $659,460 = $98,540 \text{ (net income)}$

Step 3: Use the capitalization formula to find the property’s value.

\[
\text{Income} \div \text{Rate} = \text{Value}
\]

$98,540 \div .0925 = $1,065,297$
Summary

Capitalization Problems

Value $\times$ Capitalization Rate = Net Income

- Capitalization rate: the rate of return an investor would want from the property.
- The higher the cap rate, the lower the value.
- Subtract operating expenses from gross income to determine net income.
- OER: Operating expense ratio
Tax Assessment Problems

Tax assessment problems are another type of percentage problem.

Whole $\times$ % = Part

Assessed Value $\times$ Tax Rate = Tax
Tax Assessment Problems

Assessment ratio

- Some problems simply give you the assessed value.
- Others give you the market value and the assessment ratio, and you have to calculate the assessed value.

Example: The property’s market value is $100,000 and the assessment ratio is 80%.

$100,000 \times 0.80 = 80,000$

The assessed value is $80,000.
Example: The property’s market value is $200,000. It is subject to a 25% assessment ratio and an annual tax rate of 2.5%. How much is the annual tax the property owner must pay?
Tax Assessment Problems

Assessment ratio

- Step 1: Calculate the assessed value by multiplying the market value by the ratio.
  
  \[ \text{Assessed Value} = \text{Market Value} \times \text{Ratio} \]
  
  \[ \$200,000 \times 0.25 = \$50,000 \text{ (assessed value)} \]

- Step 2: Calculate the tax.

  \[ \text{Assessed Value} \times \text{Tax Rate} = \text{Tax} \]
  
  \[ \$50,000 \times 0.025 = \$1,250 \text{ (tax)} \]

The property owner is required to pay $1,250.
Tax Assessment Problems

Tax rate per $100 or $1,000

- In some questions, the tax rate will not be expressed as a percentage, but as a dollar amount per hundred dollars or per thousand dollars of assessed value.

- Divide the value by 100 or 1,000 to find the number of $100 or $1,000 increments. Then multiply that number by the tax rate.
Example: A property is assessed at $125,000. The tax rate is $2.10 per hundred dollars of assessed value. What is the annual tax?
Tax Assessment Problems

Tax rate per $100

- **Step 1:** Determine how many hundred-dollar increments are in the assessed value.
  
  $125,000 \div 100 = 1,250 \ ($100 \ increments)$

- **Step 2:** Multiply the number of increments by the tax rate.
  
  $1,250 \times \$2.10 = \$2,625 \ (annual \ tax)$
Example: A property is assessed at $396,000. The tax rate is $14.25 per thousand dollars of assessed value. What is the annual tax?
Tax Assessment Problems

Tax rate per $1,000

- Step 1: Determine how many thousand-dollar increments are in the assessed value.
  $396,000 ÷ 1,000 = 396 ($1,000 increments)

- Step 2: Multiply the number of increments by the tax rate.
  \[ 396 \times $14.25 = $5,643 \] (annual tax)
One other way in which a tax rate may be expressed is in terms of mills per dollar of assessed value.

- A mill is one-tenth of a cent, or one-thousandth of a dollar.
- To convert mills to a percentage rate, divide by 1,000.
Tax Assessment Problems

Tax rate in mills

Example: A property is assessed at $290,000 and the tax rate is 23 mills per dollar of assessed value. What is the annual tax?
Step 1: Convert mills to a percentage rate.

\[
23 \text{ mills/dollar} \div 1,000 = .023 \text{ or 2.3%}
\]

Step 2: Multiply the assessed value by the tax rate to determine the tax.

\[
$290,000 \times .023 = $6,670
\]
Summary

Tax Assessment Problems

Assessed Value × Tax Rate = Tax

- To find assessed value, you may have to multiply market value by the assessment ratio.
- Tax rate may be given as a percentage, as a dollar amount per $100 or $1,000 of value, or in mills.
- Divide mills by 1,000 to get a percentage rate.
Seller’s Net Problems

- This type of problem asks how much a seller will have to sell the property for to get a specified net amount from the sale.
In the basic version of this type of problem, you’re told the seller’s desired net and the costs of sale.

Start with the desired net proceeds, then:

1. add the costs of the sale, except for the commission
2. subtract the commission rate from 100%
3. divide the results of Step 1 by the results of Step 2
Example: A seller wants to net $220,000 from the sale of his property. He will pay $1,650 in attorney’s fees, $700 for the escrow fee, $550 for repairs, and a 6% brokerage commission. How much will he have to sell the property for?
1. Add the costs of the sale to the desired net:
   $220,000 + $1,650 + $700 + $550 = $222,900

2. Subtract the commission rate from 100%:
   100% - 6% = 94%, or .94

3. Calculate the necessary sales price:
   $222,900 ÷ .94 = $237,127.66

The sales price will have to be at least $237,130 for the seller to get his desired net.
There are some variations on this type of problem.

Variation 1: You’re told the original purchase price and the percentage of profit the seller wants from the sale.

- This requires an additional step, calculating the seller’s desired net.
Example: A seller bought land two years ago for $72,000 and wants to sell it for a 25% profit. She’ll have to pay a 7% brokerage fee, $250 for a survey, and $2,100 in other closing costs. For what price will she have to sell the property?
1. Use the “Then and Now” formula to calculate the desired net.

Then × Rate = Now

$72,000 \times 1.25 = $90,000 desired net

Or calculate the profit and add it to the original value to get the desired net:

$72,000 \times 25\% = $18,000 + $72,000 = $90,000
2. Next, add the costs of sale, except for the commission.
   
   $90,000 + $250 + $2,100 = $92,350

3. Subtract the commission rate from 100%.
   
   100% - 7% = 93%, or .93

4. Finally, calculate the necessary sales price.
   
   $92,350 ÷ .93 = $99,301
In another variation on this type of problem, you’re asked to factor in the seller’s mortgage balance.

- This is more realistic, since most sellers have a loan to pay off.
- Just add the loan balance as one of the closing costs.
Example: A seller wants to net $24,000 from selling his home. He will have to pay $3,300 in closing costs, $1,600 in discount points, $1,475 for repairs, $200 in attorney’s fees, and a 6% commission. He will also have to pay off the mortgage balance, which is $46,050. How much does he need to sell his home for?
Seller’s Net Problems
Variation 2

1. Add the costs of sale and the mortgage balance to the desired net.
   
   \[ \$24,000 + \$3,300 + \$1,600 + \$1,475 + \$200 + \$46,050 = \$76,625 \]

2. Subtract the commission rate from 100%.
   
   \[ 100\% - 6\% = 94\%, \text{ or } .94 \]

3. Finally, calculate the necessary sales price.
   
   \[ \$76,625 \div .94 = \$81,516 \]
Summary

Seller’s Net Problems

1. Desired Net + Costs of Sale + Loan Payoff
2. Subtract commission rate from 100%
3. Divide Step 1 total by Step 2 rate. Result is how much property must sell for.
Proration Problems

Prorating an expense means dividing it proportionally, when someone is responsible for only part of it.

Items often prorated in real estate transactions include:

- property taxes
- insurance premiums
- mortgage interest
Proration Problems

Closing date is proration date

- Seller’s responsibility for certain expenses ends on closing date.
- Buyer’s responsibility for certain expenses begins on closing date.
Proration Problems

In advance or in arrears

- If seller is in arrears on a particular expense, seller will be charged (or debited) for a share of the expense at closing.
  - Buyer may be credited with same amount.

- If seller has paid an expense in advance, seller will be refunded a share of the overpaid amount at closing.
  - Buyer may be debited for same amount.
Proration Problems
365 days or 360 days

You will be told whether to use a 365-day or 360-day year.

- In a 365-day year, use the exact number of days in each month.
- In a 360-day year, each month has 30 days.
Proration Problems

3 Steps

Prorating an expense is a three-step process:

1. Calculate the per diem (daily) rate of the expense.
2. Determine the number of days the party is responsible for.
3. Multiply per diem rate by number of days.
Proration Problems

Property taxes

- Remember that in some states, the property tax year is different from the calendar year.

- Also, payments are sometimes divided into installments.
Example: The closing date is Feb. 3 and the seller has already paid the annual property taxes of $2,045. At closing, the tax amount for Feb. 3 – June 30 (tax year begins on July 1) will be a credit for the seller and a debit for the buyer. How much will the buyer owe the seller for the taxes? (Base calculations on a 360-day year and 30-day months).
Proration Problems

Property taxes

Step 1: Calculate the per diem rate.

$2,045 \div 360 = $5.68

Step 2: Count the number of days.

28 (Feb.) + 30 (Mar.) + 30 (Apr.)
+ 30 (May) + 30 (Jun.) = 148 days

Step 3: Multiply rate by number of days.

$5.68 \times 148 = $860.64

The seller will be credited $860.64 at closing. The buyer will be debited for the same amount.
Example: The sellers of a house have a one-year prepaid hazard insurance policy with an annual premium of $1,350. The policy has been paid for through March of next year, but the sale of their house will close on November 12 of this year. The buyer’s responsibility for insuring the property begins on the day of closing. How much will be refunded to the sellers at closing? (Use a 360-day year.)
Proration Problems

Insurance

Step 1: Calculate the per diem rate.

\[
\frac{1,350}{360} = 3.75
\]

Step 2: Count the number of days.

\[
19 \text{ (Nov.)} + 120 \text{ (Dec.–March)} = 139 \text{ days}
\]

Step 3: Multiply per diem rate by number of days.

\[
3.75 \times 139 = 521.25
\]

Sellers will be credited $521.25.
(Buyer will not be debited for this amount, unless she is assuming sellers’ policy.)
For interest prorations, don’t forget that mortgage interest is almost always paid:
• on a monthly basis
• in arrears (at end of the month in which it accrues)

If you aren’t given the amount of annual interest, first use the loan amount and interest rate to calculate it.
• Then do the other proration steps.
Two types of mortgage interest usually have to be prorated at closing:

- seller’s final interest payment
- buyer’s prepaid interest
Example: A seller is selling her home for $275,000. She has a mortgage at 7% interest with a balance of $212,500. The sale closes on May 14, and the seller will owe interest for the day of closing. At closing, how much will the seller’s final interest payment be? (Use a 360-day year.)
Prorating Mortgage Interest

Seller’s final interest payment

Step 1: Calculate the annual interest.
$212,500 \times .07 = $14,875

Step 2: Calculate the per diem rate.
$14,875 \div 360 = $41.32

Step 3: Count the number of days.
May 1 through May 14 = 14 days

Step 4: Multiply per diem by number of days.
$41.32 \times 14 = $578.48
Prepaid interest: At closing, the buyer is charged interest for closing date through the end of the month in which closing occurs. Also called interim interest.

- Example: Sale is closing on April 8.
  - Buyer’s first loan payment, due June 1, will include May interest, but not April interest.
  - At closing, buyer will pay interest for April 8 through April 30.
Prorating Mortgage Interest

Buyer’s prepaid interest

Example: A buyer purchased a house with a $350,000 loan at 5.5% annual interest. The transaction closes Jan. 17. The buyer is responsible for the day of closing. How much prepaid interest will the buyer have to pay? (Use a 360-day year.)
Buyer’s prepaid interest:

Step 1: Calculate the annual interest.

\[ \$350,000 \times 0.055 = \$19,250 \]

Step 2: Calculate the per diem rate.

\[ \$19,250 \div 360 = \$53.47 \]

Step 3: Count the number of days.

Jan. 17 through Jan. 30 = 14 days

Step 4: Multiply per diem rate by days.

\[ \$53.47 \times 14 = \$748.58 \]

Buyer will owe $748.58 in prepaid interest at closing.
Summary
Proration Problems

1. Calculate per diem rate.  
   (365-day or 360-day year)
2. Count number of days.
3. Multiply per diem rate by number of days.